Investigation on Frequency Responses for Supersonic Intake Based on High Resolution Simulation

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Abstract
The terminal shock within the supersonic inlet is subjected to both internal and external perturbations during the operation of supersonic air-breathing engine. In order to improve the performance of air breathing engine and prevent inlet unstart, it is necessary to control the terminal shock during various disturbance, and determine the dynamics of intake which is valuable for the controller. To simulate the dynamics of intake, high resolution simulation of supersonic inlet based on high order weighted essentially non-oscillatory schemes (WENO) was conducted. To find the interactions of inlet to engine and external conditions, the nonlinear inlet simulation model was subjected to sinusoidal perturbations and the response of shock and static pressure along the inlet in time domain to back pressure and upstream Mach number were obtained and compared to get the dynamics of inlet. Furthermore, the data in time domain were exploited by least-square curve fitting to generate the magnitude ratio and phase shift. Furthermore, according to frequency sweeps, frequency responses of shock position and that of static pressure throughout the intake to back pressure up to 1500Hz and free stream Mach number up to 100Hz have been obtained. The results show that frequency response of shock to back pressure is quite different from that to upstream mach number, and the amplitude ratio of static pressure upstream of shock descends with the growth of frequency, while that of pressure downstream of shock reach the peak at several frequencies, which is induced by the movement of shock.

Keywords
Supersonic Intake; Intake Dynamics; WENO; Frequency Responses

Introduction
Ramjet technology has evolved in more than hundred years, during which supersonic intake has significant effluence on the maturation of ramjet propulsion[1]. For mixed-compression inlets, optimum internal performance is provided by maintaining the terminal shock near the inlet throat[2-7], which provides higher pressure recovery and reduces flow distortion at the outlet[6]. However, disturbances may occur either upstream or downstream of inlet[2-18]. As shown in Fig1, the terminal shock responds to both upstream from free airstream and downstream disturbances from combustor or compressor, resulting in inlet unstart if terminal shock moves upstream from the throat[9-11]. Unstart causes a sharp reduction in mass flow, pressure recovery and large increase of drag[6-11], what’s more, inlet buzz, combustor blowout may occur, so it is valuable to reject unstart, which demands active control of the inlet to keep a stable shock position in the presence of random airflow disturbance and prevent inlet unstart[2,7-10,15].

It has attracted much attention to the dynamics of shock motions[9-18]. A low-order model that captures the key features of the shock motions is invaluable for the development of control strategies[9] and the evaluating such a
control[10]. Previous studies have shown that the dynamic model of shock motions oriented for control is established by linearization[9,16], such as theoretical analysis[16], data fitting[10,14,18], or frequency identification[3,17]. The modeling based on linearization is convenient in application. However, it is only suitable to intake flow with small perturbations[6], but unable to represent nonlinearities. Finite difference methods[6,18] can work with variable time step and present time-variant properties with higher resolution[18].

When the flow fields involve shock waves, the numerical schemes should be essentially oscillation free near the discontinuities. In recent years, many efforts have been devoted to the development of high resolution shock-capturing schemes that are higher order accurate in the smooth regions. One class of the numerical schemes among them are higher order ENO and WENO schemes[19-24]. The high resolution simulation code has been completed and verified in Ref [18], the transient responses of shock motion under perturbations coming from free stream and combustor have been calculated and studied, the results show that with the increase of magnitude of the disturbance, the nonlinearity of shock motion becomes more significant. All the analysis of [18] was on time domain responses of shock, and the frequency responses will be presented in this paper to provide a fast, flexible and high accurate method to model the dynamics of intake, which will be useful for the shock position controller design.

The paper is organized as follows. Firstly, the back ground and the state of art of frequency dynamics of supersonic intake were presented. Secondly, the high resolution simulation model was described. Besides, the dynamics of shock and static pressure within the intake in time domain under different frequencies were presented and analyzed. Furthermore, frequency response of intake to back pressure and upstream Mach number were obtained by frequency sweep and least-square curve fitting. Finally, the conclusion is summarized and remarked.

Mathematical Model

Governing Equations

In this section, the basic governing equations for solving the one dimensional gas behavior with variable area are presented, and the numerical schemes are simply described to get the gas dynamics. If the cross-sectional area of a flow passage varies very slowly and the radius of curvature of the central axis of the passage is large contrasted to the passage height, the flow inside the passage is said to be a quasi-one-dimensional flow[6]. In this case, the flow properties are assumed to be uniform across all surfaces perpendicular to the mean flow direction. The proper governing equations are as follows.

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = J$$

(1)

which is equivalent to

$$\begin{align*}
\frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho u A)}{\partial x} &= 0 \\
\frac{\partial (\rho u A)}{\partial t} + \frac{\partial (\rho u^2 A + p A)}{\partial x} &= p \frac{\partial A}{\partial x} \\
\frac{\partial (\rho E)}{\partial t} + \frac{\partial (u \rho A H)}{\partial x} &= 0
\end{align*}$$

(2)

where $E = \epsilon + 0.5u^2$ is the total internal energy, $\epsilon$ is the internal energy, $H = h + 0.5u^2$ is total internal enthalpy, $h$ is the enthalpy. $A$ is the area of inlet, $u$ is the velocity, $\rho$ is the density, $p$ is the static pressure. $U=[\rho,\rho u,\rho u^2+p,\rho A,\rho E]^T$ is the vector of conservative variables, and the flux is $F=[\rho u A, \rho u^2 A + p A, u \rho A H]^T$, $J$ is the source term considering the area variation. It is assumed perfect gas thermodynamics, therefore

$$p = (\gamma-1) \rho (E-0.5u^2)$$

$$a = \sqrt{\gamma p/\rho}$$

(3)

where $a$ is the speed of sound, $\gamma$ is the specific heat ratio, for air $\gamma=1.4$. 

Dimensionless Governing Equations

In this section, the governing equations are non-dimensionalized by using the density, pressure and static temperature as reference density, pressure and temperature, the intake length as the reference length, the throat area as the reference area, and the \( u_f = \sqrt{\frac{p_m}{\rho_m}} \) as the reference velocity. So the dimensionless dynamic equations of motion (continuity, momentum, and energy) are shown as follows

\[
\frac{\partial \tilde{U}}{\partial t} + \frac{\partial \tilde{F}}{\partial \tilde{x}} = \tilde{J}
\]

which is equivalent to

\[
\begin{align*}
\frac{\partial (\tilde{\rho} \tilde{\lambda})}{\partial t} + \frac{\partial (\tilde{\rho} \tilde{u} \tilde{\lambda})}{\partial \tilde{x}} &= 0 \\
\frac{\partial (\tilde{\rho} \tilde{u} \tilde{\lambda})}{\partial t} + \frac{\partial (\tilde{\rho} \tilde{u}^2 \tilde{\lambda} + \tilde{p} \tilde{\lambda})}{\partial \tilde{x}} &= \tilde{p} \frac{\partial \tilde{\lambda}}{\partial \tilde{x}} \\
\frac{\partial (\tilde{\rho} \tilde{E} \tilde{\lambda})}{\partial t} + \frac{\partial (\tilde{u} \tilde{p} \tilde{A} \tilde{\lambda})}{\partial \tilde{x}} &= 0
\end{align*}
\]

The dimensionless perfect gas equations of state and the sonic velocity are as follows

\[
\tilde{p} = \tilde{\rho} \tilde{T} = (\gamma - 1) \tilde{\rho} \left( \tilde{E} - 0.5 \tilde{u}^2 \right) \\
\tilde{a} = \sqrt{\gamma \tilde{p} / \tilde{\rho}}
\]

Especially, if the area of the duct is constant, the above equations are equivalent to the one dimensional Euler equations.

Numerical Method

1) WENO Schemes

The fifth-order Finite different scheme is adopted to simulate the shock motion in supersonic intake. When solving the Euler equations, the evaluation of the numerical flux functions for the characteristic-wise finite different WENO scheme involves the following steps:

At each fixed \( \tilde{x}_{j+0.5} \), the average state \( \tilde{U}_{j+0.5} \) is computed by the Roe average.

\[
\tilde{U}_{j+0.5} = R^{\text{Roe}} \left( \tilde{U}_{j+1}, \tilde{U}_j \right)
\]

The eigenvalues \( \tilde{\lambda}_{j+0.5}^i \ (i = 1, 2, 3) \) and the left eigenvectors \( \tilde{l}_{j+0.5}^i \ (i = 1, 2, 3) \) and the right eigenvectors of Jacobin \( F' \left( \tilde{U}_{j+0.5} \right) \) are computed in terms of \( \tilde{U}_{j+0.5} \).

\[
\begin{align*}
R &= R \left( \tilde{U}_{j+0.5} \right) \\
L &= L \left( \tilde{U}_{j+0.5} \right) \\
\Lambda &= \Lambda \left( \tilde{U}_{j+0.5} \right)
\end{align*}
\]

Based on the left eigenmatrix \( L \), the fluxes are transformed into local characteristic flow field using \( V_n = L \tilde{F}_n \). The local characteristic decompositions of the flux functions at \( x_m(m = j+1-k, j+k) \) are computed using

\[
\begin{align*}
w_n' &= \tilde{l}_{j+0.5}^i \tilde{F}_n, i = 1, 2, 3 \\
m &= j - k + 1, \ldots, j + k
\end{align*}
\]

In the local characteristic fields every component is reconstructed by WENO to generate \( \tilde{W}_{j+0.5} \) for obtaining the flux \( \tilde{F}_{j+0.5} \). The flux \( \tilde{W}_{j+0.5} \) is transformed back into physical space using
\[
\hat{F}_{j+0.5} = R \hat{\nabla}_{j+0.5}
\]

The Roe type characteristic-wise WENO scheme is less dissipative and thus achieves higher resolution than the WENO scheme based on the flux vector splitting, especially in capturing the contact discontinuities and shear layers in viscous flows[24]. But the Roe type WENO scheme admits rarefaction shocks that do not satisfy the entropy condition, therefore, certain entropy fix procedure is needed. "H-correction" procedure is adopted to calculate every component of \( \hat{\nabla}_{j+0.5} \), which is shown as follows.

\[
w_{i,WENO}^{j+0.5} = \begin{cases} 
\hat{\nabla}_{i,WENO-Roe}^{j+0.5} , & \text{min} \left( \left| \hat{F}_{j+0.5} \right| \right) \geq \eta_{j+0.5} \\
\hat{\nabla}_{i,WENO-LF}^{j+0.5} , & \text{min} \left( \left| \hat{F}_{j+0.5} \right| \right) < \eta_{j+0.5}
\end{cases}
\]

In the above equation, \( \eta_{j+0.5} \) is determined by

\[
\eta_{j+0.5} = \left[ \left| \hat{\nabla}_{j+1} - \hat{\nabla}_{j} \right| + \left| \hat{\nabla}_{j} - \hat{\nabla}_{j-1} \right| \right]^{-1}
\]

Every component of characteristic field is calculated as follows:

\[
\hat{\nabla}_{i,WENO-Roe}^{j+0.5} = \begin{cases} 
\hat{\nabla}_{i}^{j+0.5}, & \hat{\nabla}_{i}^{j+0.5} \geq 0 \\
\hat{\nabla}_{i}^{-j+0.5}, & \hat{\nabla}_{i}^{-j+0.5} \geq 0
\end{cases}
\]

where the reconstruction of \( \hat{\nabla}_{j+0.5}^{+} \) and \( \hat{\nabla}_{j+0.5}^{-} \) is based on the characteristic flux \( \hat{\nabla}_{i} \). While the LF flux splitting finite different method is

\[
\hat{\nabla}_{i,WENO-LF}^{j+0.5} = \hat{\nabla}_{i}^{j+0.5} + \hat{\nabla}_{i}^{-j+0.5}
\]

where the reconstruction of \( \hat{\nabla}_{j+0.5}^{+} \) and \( \hat{\nabla}_{j+0.5}^{-} \) is based on the positive flux and the negative flux.

The high order of WENO attributes to the convex combination of all of the candidate stencils \( \hat{\nabla}_{j+0.5} \). For 5 order WENO, \( k=3 \), suppose the \( k \) candidate stencils

\[
S_{r}(i) = \left\{ x_{j-r}, \ldots, x_{j-r+k} \right\}_{r=0, \ldots, k-1}
\]

produce \( k \) different reconstructions to the value \( \hat{\nabla}_{j+0.5} \)

\[
\hat{\nabla}_{j+0.5}^{r} = \sum_{m=0}^{k-1} c_{rm} \hat{\nabla}_{j-r+r}^{r} \quad r=0, \ldots, k-1
\]

The coefficient \( c_{rm} \) can be found in Ref [19] and [22]. WENO reconstruction takes a convex combination of all \( \hat{\nabla}_{j+0.5}^{r} \) as a new approximation to the cell boundary value \( \hat{\nabla}_{j+0.5}^{+} \).

\[
\hat{\nabla}_{j+0.5}^{+} = \sum_{r=0}^{k-1} \omega_{r} \hat{\nabla}_{j+0.5}^{r}
\]

The weight coefficient \( \omega_{r} \) is the key to the success of WENO. For Jiang-Shu[20] nonlinear weights, the weights are defined as

\[
\omega_{r} = \frac{\alpha_{k}}{\sum_{l=0}^{k-1} \alpha_{l}} = \frac{d_{k}}{\left( \beta_{k} + e \right)^{p}}
\]
The coefficient $d_k$ are the optimal weights, are given by

$$d_0=0.3, d_1=0.6, d_2=0.1$$  \hspace{1cm} (19)

$\beta_k$ is the smoothness indicator of WENO. For WENO-Z scheme, it can be found in Ref [21].

2) **Source Term**

The source term due to geometry is discreted by fourth order central difference

$$\left( \frac{\partial A}{\partial x} \right)_j = \frac{8(A_{j+1}-A_{j}) - (A_{j+2}-A_{j+1})}{12\Delta x}$$  \hspace{1cm} (20)

3) **Boundary Condition**

The inlet plane of intake is supersonic, so the parameter of upstream boundary is given by free stream condition. The outlet is subsonic, with given pressure, so the other parameters of outlet are gotten by extrapolating the internal flow information. What’s more, near the boundary, the flux reconstruction may use the ghost nodes, in this case, the weights of stencil with ghost nodes are set to zero. It may reduce the order, but it is very robust.

4) **Time Integration**

In present work, the time integration is performed by means of a three-stage, TVD Runge-Kutta schemes. Defining the semidiscreted form as

$$L_j(v) = -\frac{1}{\Delta t} (\hat{f}_{j+1/2} - \hat{f}_{j-1/2})$$  \hspace{1cm} (21)

Then this schemes is given by

$$v_j^{(0)} = v_j^{(1)} + \Delta t L_j(v_j^{(1)})$$

$$v_j^{(2)} = \frac{3}{4} v_j^{(1)} + \frac{1}{4} v_j^{(0)} + \frac{1}{4} \Delta t L_j(v_j^{(1)})$$

$$v_j^{(n+1)} = \frac{1}{3} v_j^{(n)} + \frac{2}{3} v_j^{(2)} + \frac{2}{3} \Delta t L_j(v_j^{(2)})$$  \hspace{1cm} (22)

In order to keep the high accuracy, the time step is determined by

$$\Delta t = \sigma \left( \frac{\Delta x}{\max_j \left| u_j \right| + a_j} \right)^{5/3} \hspace{1cm} (23)$$

where $\sigma$ is the courant number, $0<\sigma<1$, the solution is stable. In order to obtain constant output time interval, the final step was slightly adjusted.

5) **Shock Detector**

In order to analysis the dynamics of shock position, the key to success is the shock detector. The most simple method is that possible method is to use the pressure variation just like MacCormack scheme with

$$q_j = \left| \frac{p_{j+1} - 2p_j + p_{j-1}}{p_{j+1} + 2p_j + p_{j-1}} \right|$$  \hspace{1cm} (24)

In the smooth area, the parameter $q_j$ is

$$q_j = \left| \frac{\partial^2 p}{\partial x^2} (\Delta x)^2 + O((\Delta x)^4)}{4p_j + O((\Delta x)^2)} \right|$$  \hspace{1cm} (25)

so the $q_j$ is very small. While near the shock, $q_j$ is very large. In this case, the peak position is defined as shock
front. If this position was defined as shock position, as shown in Ref [18], the shock position discontinuities which was caused by the space step of the grid point. In reality, the motion of shock continues, so the interpolation around the shock front area is utilized to obtain shock position, which is defined as the location where the Mach number is one.

Results and Discussion

Dynamic Response Analysis

In this paper, the geometry of the supersonic inlet and the boundary condition are the same as Ref [18]. For clarity, it is shown again. The dimensionless area of intake varies with \( \tilde{x} \) only:

\[
\tilde{A} = \begin{cases} 
1 + \left( \tilde{A}_{\text{in}} - 1 \right) \left( 25(x - 0.2)^2 \right), & 0 < \tilde{x} \leq 0.2 \\
1 + \left( \tilde{A}_{\text{e}} - 1 \right) \left( 1.5(1.25x - 0.25)^2 - 0.5(1.25x - 0.25)^4 \right), & 0.2 < \tilde{x} \leq 1
\end{cases}
\]

(29)

where \( \tilde{A} \) is the ratio of inlet area to the throat area and given as 1.1, and \( \tilde{A}_{\text{e}} \) is the ratio of outlet area to the throat area and specified as 2. The free stream Ma number is selected as 2.5, the initial shock position is specified as 0.6.

In this section, the numerical solution has been computed on 100 uniform grids using a courant number 0.5 at timespan of 20ms. The upstream static temperature is taken to be 300K, the total inlet length is 2m. The simulation is done on dimensionless algorithm and has been verified in Ref[18].

The static pressures at \( \tilde{x} \)=0.3 and 0.7 are recorded to give the dynamic response of pressure, with simplicity the two points are numbered as U and D along the flow direction. Point U locates in the upstream of initial shock position, and point D in the downstream of initial shock position. The shock position is identified by combination of shock detector and interpolation method, which means the discrepancy of shock position and theritical ones with one space interval.

The responses of inlet to back pressure and upstream Mach number with relative magnitude lower than 10% were linear, and thereby the relative disturbance magnitude was set to 5%. The dynamic response of shock position to the symmetrical sinusoidal back pressure with frequency at 10Hz, 20Hz and 40Hz were calculated, and the numerical results are plotted in Fig2.

![FIG.2 TIME DOMAIN RESPONSE OF SHOCK POSITION TO BACK PRESSURE AT DIFFERENT FREQUENCIES](image)

As shown in Fig2, after a short dead time, shock starts to move upstream with the increase of back pressure, the shock moves between 0.54 and 0.66, which means it can’t reach Point U or D. Besides, after one period the wave shape is nearly similar to each other with the same peak values. The peak shock position at 40Hz is slightly larger than that at 10Hz and 20Hz, which suggests that there may be a resonance roughly at 40Hz. The shock moves smoothly, which implies the effectiveness of the improvement of shock detector. It is clear that the period of shock shortens with the increase of the exciting frequency.
The dynamic responses of shock position to the symmetrical sinusoidal upstream Mach number with frequency at 10Hz, 20Hz and 40Hz were computed, and the numerical results are presented in Fig3.

As shown in Fig3, after a very short dead time, shock moves downstream with the increase of upstream Mach number, the shock moves between 0.5 and 0.7, which means it can’t reach Point U or D. The wave shape are nearly identical to each other during each period, with the same peak values. The peak shock position at 40Hz is slightly larger than that at 10Hz and 20Hz, and the peaks are roughly the same at lower frequency. The period of shock position decreases as the frequency of disturbance grows.

**Frequency Response of Supersonic Intake**

The high resolution simulation model of supersonic inlet was perturbed by upstream Mach number or back pressure with the shape of sin wave, so the dynamic response can be generated, however, it is difficult to get the amplitude ratio and phase shift, which is important for frequency responses. In order to solve this problem, the curve fitting based on least square method was utilized to determine the frequency characteristics, namely the frequency responses of supersonic intake to upstream or downstream perturbations. In order to capture the frequency characteristic lower than 150Hz, the sample frequency must be larger than 300Hz, in this paper, the sample frequency is 1000Hz. Furthermore, the computed time is nealy at 0.001 ms, so it’s very robust to temporal resolution and space resolution.

**Back Pressure Perturbation**

The frequency response results of shock to back pressure at frequency up to 150Hz are plotted in Fig4, and the amplitude curve is plotted as normalized amplitude ratios which are normalized by dividing by the amplitude ratio at 1Hz. It is clearly there is a resonance at about 49Hz. The phase lag increases as the frequency grows. In addition, the frequency response of shock to back pressure is very similar to that of lag element.
It is clear from Fig3, shock can’t move forward to point U, which is located in fully supersonic regime, and is only affected by upstream disturbance, so the static pressure at Point D is given in Fig5, the magnitude ratio of static pressure at D nearly keeps constant when frequency is lower than 10, and then surges as frequency goes up, and then reaches the peak at 55Hz, the resonance of static pressure downstream of shock is very similar to that of the shock. The difference may lie in that the downstream of shock is subsonic and affected by both the downstream perturbation and the upstream movement of shock. It is clear that the upstream of shock is supersonic flow and can’t be affected by back pressure disturbance at outlet, and the flow which changes with downstream disturbance is located between outlet and shock, so it forms a closed chamber with closed ends and in this condition it is easy to generate resonance.

**Upstream Mach Number Perturbations**

The calculated results of frequency response of shock position to upstream Mach number are shown in Fig6. It is clearly there are several peaks at 6Hz, 26Hz, 48Hz, and 64Hz, which implies that the dynamics may be represented by the distributed system. The phase lag is smaller that of shock to back pressure, in that it needs shorter time for disturbance propagating from inlet plane to shock during which the velocity of characteristic waves is very large in supersonic duct.

The magnitude of each response rises with the growth of frequency for pressure located at U and D, and it changes slowly with frequency less than 10Hz, however, with the growth of frequency, the magnitude of pressure response at point D has more peaks, while that at point U is quite different, it grows as frequency increases. The difference may lie in that point U is only affected by upstream as shown in Fig3 (shock can’t reach point U), while point D is affected by both downstream and upstream. The magnitude curve of static pressure at D is very complex coupled
with more peaks than that of shock position. As showed in Fig7, the phase lag grows with the growth of frequency, and the difference between that of pressure at U and D is very small.

![Graph]

FIG.7 FREQUENCY RESPONSE OF STATIC PRESSURE WITHIN THE INLET TO UPSTREAM MACH NUMBER

Conclusions

The frequency dynamics of shock and static pressure throughout the supersonic intake are investigated by perturbing the high resolution simulation model, and the conclusions are summarized as follows.

The frequency of shock movement increases with the growth of the frequency of the external disturbance. The amplitude ratio of shock to back pressure and upstream Mach number changes slowly at frequency lower than 20Hz.

The frequency response of shock to upstream Mach number is quite different from that to back pressure with more peaks in the magnitude curve and smaller phase lag.

The frequency response of static pressure downstream of shock to back pressure is quite similar to that of shock, which can be represented by lag element.

The frequency responses are very complex, so in the future the rational polynomial transfer function approximation may be introduced by frequency identification, and be reduced by the model reduction to get low order transfer function for controller design.

REFERENCES


