A Coupled Finite Element-circuit Analysis on Flexible PVDF Energy Harvester

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Abstract

Piezoelectric energy harvester (PEH) has attracted intensive attention due to its simple configuration, high power density and no need for extra power supply, and in particular, poly (vinylidene fluoride) (PVDF)-based PEH additionally shows flexibility and exhibiting promises in applications such as the power supply for wearable devices. However, the actual PVDF-PEH produces much lower performance than theoretical estimations. This is because on one hand the existing theoretical calculations are based on over-simplified models, on the other hand, the optimum resistive load of PVDF-PEH is too high to match in practical applications. In this paper, a coupled finite-element-circuit analysis is conducted for PVDF-PEH to give accurate simulation for the device performance. Besides, multi-layer PVDF membranes are used in our model which notably decrease the optimum resistive load. Phase difference between output voltage and mechanical z-displacement is studied: the external load meets the optimum value when the phase difference is 135°, providing an alternative insight into the load matching problem. Finally we have simulated an optimized device with four 50μm thick PVDF layers, which shows an output power density of 2.69mW/cm² under 47.6Hz, 0.5g sinusoidal vibration and 10.76mW/cm² under 47.6Hz, 1.0g sinusoidal vibration.

Keywords

PVDF; Piezoelectric Energy Harvesting; Coupled Finite Element-circuit Analysis

Introduction

The technique of Internet of Things develops rapidly in recent years, attached with it, wireless sensor is one of the hot spot for researches. It is difficult for wireless sensors to replace batteries because of the huge number, widespread distribution and remote location. Implanted devices have other difficulties such as it is dangerous to replace batteries for cardiac pacemaker. However, with the development of microelectronic technology, the consumption of volume and energy decreases greatly. It is possible for electronic devices to collect energy from the outside environment. Most of the energy around us is left unused such as wind, solar, electromagnetic radiation, mechanical vibration and temperature fluctuation. Compared with others, the mechanical structure processes high energy density and it is relatively easy to collect. In this paper, a piezoelectric method is adopted because of its simple structure, high output voltage and no need for extra power supply.

Piezoelectric ceramics (e.g. PZT) and piezoelectric single crystal (e.g. PMN-PT) are usually adopted in piezoelectric energy harvester (PEH) as the piezoelectric constant is high. However, recently studies indicate that it is the figure of merit (d,e.g, the product of piezoelectric strain constant and piezoelectric voltage constant) of piezoelectric materials that determines the output power of PEH in non-resonant mode. Poly (vinylidene fluoride) (PVDF) is piezoelectric material which processes high figure of merit compared with PZT. Besides, PVDF is environmental friendly (no lead), rather tough, stable in environment and easy to process and model. As PVDF have low elastic modulus, its first resonance frequency is low, which is appreciated in actual use. The question is how PVDF PEH performs under resonant condition.

People have used PVDF in energy harvester for a long time. In 1998, Kymissis et al have put PVDF into the shoes, collecting energy from walking. From then on, Vatansever et al, Li et al, Taylor et al and Akaydin et al put PVDF membranes into air flow or water to collect energy from them. However, PVDF in these attempts works in non-resonant mode while the efficiency of energy collection can be enhanced in resonant mode. Later, Roundy et al gave the resonant mode output power for PVDF PEH using a theoretical way: the output power of the device (less than 1cm³ volume) can be as high as 200μW under 120Hz, 0.25g vibration excitation. The output power for PVDF PEH is at the same level as similar devices based on PZT. Now, PZT
based PEH has arrived at this estimation, but PVDF PEH is far away from it. Jiang et al fabricated their PVDF cantilever PEH (25mm×16mm×5mm), showing an output power of 16μW under 1.2g sinusoidal vibration excitation. Cao et al fabricated their PVDF PEH (5.1mm × 19.1mm × 0.5mm), showing an output power of 2.4μW under 120Hz, 0.5g sinusoidal vibration excitation.

There are usually three categories for PEH simulation: lumped parameter model, distributed parameter model and Rayleigh approximate distribution parameter model. The accuracy for lumped parameter model is low and Roundy et al’s model is based on this. Elvin et al adopted a Rayleigh approximate distribution parameter model, using finite element software and circuit analysis software respectively to solve mechanical parts and circuits. The simulation results meet the experimental one well. Almost at the same time, Zhu et al brought up their coupled finite element – circuit model, connecting directly using circuit element in ANSYS and got a good result. But these work is limited to the validation of this method and has not been used on PVDF based PEH. In this paper, we used a coupled finite element – circuit method to simulate PVDF based PEH, hoping to get the characters on PVDF PEH and express why the actual performance is much lower than Roundy et al’s estimation.

**Piezoelectric Cantilever Model**

**Theory**

When piezoelectric materials are applied to external force, corresponding potential can be motivated. Based on it, people have fabricated PEH devices. Sodano et al’s work showed that general control equation of cantilever PEH devices can be expressed as equation 1 and 2. These equations are based on Euler-Bernoulli beam assumption.

\[ M \ddot{z} + C \dot{z} + Kz - \Theta \phi = F \]  
(1)

\[ C \phi + Q + \Theta z = 0 \]  
(2)

Where \( z \) stands for displacement of piezoelectric material, \( M \) stand for mass matrix, \( K \) stands for stiffness matrix, \( C \) stands for damping matrix, \( \Theta \) stands for electromechanical coupling matrix, \( C_p \) stands for the capacitance of piezoelectric material, \( F \) stands for the external force, \( \phi \) stands for voltage potential and \( Q \) stands for transferred charge. The damping matrix is proportional with the stiffness matrix and mass matrix where \( \alpha \) and \( \beta \) are corresponding coefficient.

\[ C = \alpha M + \beta K \]  
(3)

In order to get the needed coefficient \( \alpha \) and \( \beta \), solve the equation 2-4.

\[
\begin{bmatrix}
1/2\omega_1 & \omega_1/2 & 1/2\omega_2 & \omega_2/2
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}
= \begin{bmatrix}
\bar{\xi}_1 \\
\bar{\xi}_2
\end{bmatrix}
\]  
(4)

where \( \bar{\xi} \) stands for the damping ratio at one particular angular velocity \( \omega \). After specifying the values of damping ratio and angular velocity, \( \alpha \) and \( \beta \) could be solved. As PVDF has low mechanical quality factor \( Q_m \), the damping cannot be ignored. Referring to Brown et al’s estimation, \( Q_m \) is set to be 11. The conversion relation can be expressed as equation 5.

\[ Q_m = 1/(2 \cdot \bar{\xi}) \]  
(2)

Piezoelectric module of finite element software is used to solve equation 1, while SPICE is adopted to solve equation 2. Jointing these two parts, piezoelectric energy harvesting process can be simulated accurately.

**Device Configuration and Modeling**

PVDF beam is fabricated in cantilever shape: Four layers of PVDF piezoelectric membranes are clamped. One end the beam is fixed, the other end is attached with the load mass. Sinusoidal displacement excitation is applied to the fixed end and vibration acceleration is controlled by vibration amplitude. As PVDF membrane has high flexibility, base plate is rarely adopted in order to make better use of mechanical deformation and enhance the output power. Besides, the deformation of PVDF membrane under load mass is very small compared with the deformation of other parts and this will be discussed in the next part. The material parameter and device shape are listed in table 1. PVDF PEH usually processes high optimum resistive load as the dielectric constant is much lower compared with piezoelectric ceramics. Thus, a multi-layer method is adopted to minimize this disadvantage. Four PVDF membranes are connected in parallel. The polarization direction and external circuit are illustrated in Figure 1a. When deformation occurs in PVDF, the charge accumulates in the surface. Then the electrical potential is generated under which the charge flows over the external resistive load.

On the mechanical boundary condition part, the surfaces of the cantilever’s fixed end are applied as “prescribe displacement” condition to motivate vibration. Other surfaces are left “free” condition. On
the electrical boundary condition part, the top and bottom surfaces of PVDF membranes are set as "floating potential" condition, connected in parallel. Other surfaces are set as "zero charge/symmetry" condition. In order to avoid the anisotropy caused by meshing, rectangle grid is adopted in COMSOL as Figure 1b illustrates.

**TABLE 1 MATERIAL AND STRUCTURE PARAMETERS**

<table>
<thead>
<tr>
<th>(a) piezoelectric material: PVDF</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>E</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>M</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Elastic compliance coefficient</td>
<td>s11</td>
<td>3.704</td>
<td></td>
</tr>
<tr>
<td></td>
<td>s12</td>
<td>-1.111</td>
<td></td>
</tr>
<tr>
<td>coefficient (×10⁻⁹ Pa⁻¹)</td>
<td>s13</td>
<td>-1.111</td>
<td></td>
</tr>
<tr>
<td></td>
<td>s33</td>
<td>9.630</td>
<td></td>
</tr>
<tr>
<td></td>
<td>s44</td>
<td>9.630</td>
<td></td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>p</td>
<td>1700</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d3</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d2</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Piezoelectric constant (pC/N)</td>
<td>d3</td>
<td>-32.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d3</td>
<td>-27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d4</td>
<td>-23</td>
<td></td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>eε</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Length×width×thickness (mm)</td>
<td>l×w×t</td>
<td>6×20×0.2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Load mass: tungsten</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>E</td>
<td>411</td>
<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>μ</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>ρ</td>
<td>19250</td>
<td></td>
</tr>
<tr>
<td>Length×width×thickness (mm)</td>
<td>l×w×t</td>
<td>3×20×4</td>
<td></td>
</tr>
</tbody>
</table>

**Simulation Result and Discussion**

Firstly, we studied the vibration character of PVDF PEH. Previous works indicate that the resonant frequency is greatly influenced by external load for piezoelectric ceramics based PEH. When the external load increases from zero to infinity, the resonant frequency decreases significantly and the vibration amplitude rises after reducing, while PVDF based PEH is not. The resonant frequency changes slightly with the external load. Besides, the vibration amplitude does not change much as Figure 2a illustrates. This is because the electromechanical coupling coefficient \( k_{31} \) for PVDF (0.08-0.12) is much lower than piezoelectric ceramics (0.7-0.8). There is a general relation among open circuit resonant frequency \( f_{\text{open}} \), short circuit resonant frequency \( f_{\text{short}} \) and electromechanical coupling coefficient \( k_{31} \):

\[ f_{\text{open}} = f_{\text{short}} \cdot \sqrt{1-k_{31}^2}. \]

According to this, the open circuit resonant is adopted to replace the non-zero external load resonant frequency which is very hard to get directly. When studying the influence of external load, a fixed vibration frequency excitation is adopted. Besides, the full width at half maximum (FWHM) is as much as 6Hz (13%) which is a big advantage compared with piezoelectric ceramics based PEH because a wider range of frequency response can be acquired.

Figure 2b illustrates that the peak-to-peak voltage increases while the output power decreases after increasing and the output power shows an obvious peak value as the external load increases. The power density (absolute output power divided by effective PVDF volume) of the device is as high as 2.69mW/cm³ (output power is 64.6μW) when the external load is 6.06MΩ. If the vibration excitation is increased to 1g, the output power can be as high as 10.76mW/cm³. There is a relation to estimate optimum resistive load: \( R_{\text{opt}} = 1/(2\pi \cdot f \cdot C) \) where \( f \) is resonant frequency and \( C \) is the total capacitance. When electromechanical coupling coefficient is zero, the optimum resistive load meets this equation strictly. As the \( k \) for PVDF is rather low, these two numbers are very close. According to this equation, the optimum resistive load is 24.2MΩ when using 2 layers of 100μm PVDF membrane while it decreases to one quarter when using 4 layers of 50μm PVDF membrane. If lower optimum resistive load...
is needed, thinner PVDF membrane can be adopted. The thinnest PVDF membrane available is 20μm. Using 10 layers of 20μm PVDF membrane, the optimum resistive load can be as low as 970kΩ with the same power density. It can be seen in Figure 2c that the stress in PVDF is decreasing from the fixed end to the free end. Besides, the stress is almost zero under load mass. Thus these parts should not cover electrode. If we put electrode on these parts, according to the estimation equation, the optimum resistive load will decrease to 480kΩ at the expense of some output power.

Figure 2d illustrates that when increasing the external load, the phase difference between the vibration (mechanical 2-displacements) and output voltage changes accordingly, from 90° at short circuit to 180° at open circuit. When the external load meets the optimum value, the phase difference is excitedly 135° which provides a new approach to the load match difficulty.

Lastly, we re-estimate the Roundy et al’s model using the same parameters in their papers but different approaching methods. Our simulation output power is only 4.1μW, similar with Jiang et al and Cao et al’s actual devices (2.4–16μW), but much lower than Roundy et al’s 200μW estimation. Apart from the calculation error, Roundy et al applied a very high mechanical quality factor for PVDF (same as piezoelectric ceramics), which caused this overestimation.

**Conclusion**

Based on the big gap of PVDF PEH between the theoretical calculation and the experiment results, we applied a coupled finite element – circuit method to study the PVDF PEH and accurately simulated its energy collection process. At the same time, a multi-layer structure is adopted in order to reduce optimum resistive load. Compared with double layer structure, the optimum resistive load reduces from 24.2MΩ to 6.06MΩ. If further expanded to ten layers, the matched impedance will be only 970kΩ with the device shaped and power density unchanged. Besides,
phase difference between output voltage and vibration amplitude is showed in this paper and when phase difference is 90°, the external load meets the optimum value, showing a new approach to solve load match difficulty problem. Finally, the PVDF PEH uses four 50μm PVDF membranes and the power density is 2.69mW/cm³ under 47.6Hz, 0.5g sinusoidal vibration excitation and 6.06 MΩ external load. When the vibration acceleration is up to 1.0g, the power density is as high as 10.76mW/cm³ with other conditions unchanged.

REFERENCE


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