Seismic Performance Analysis of Frame and Frame-shear Wall Structures Based on Energy Balance

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Abstract: Based on the principle of energy analysis, we introduced the expression of energy relations of single degree-of-freedom system and the expression of energy response of multi-degree-of-freedom system. Then the model source is introduced, and the basic parameters of the model are explained. Finally, the finite element analysis of the model was carried out. The parameters of the structure were obtained by using the traditional response spectrum. The capability of the model was analyzed by MIDAS finite element software. The performance of the model under the frequent earthquakes and rare earthquakes was analyzed. The final results were deduced in the way of energy.

Keywords: energy balance, frame-shear structure, seismic performance

1. Introduction

In the 1950s, Housner first proposed energy-based seismic design of structural thinking. He tried to use the energy analysis method to study the seismic response of the structure, that the structure of the earthquake response can be seen as a seismic energy input and dissipation process, as long as the damping and hysteretic energy dissipation capacity of the structure is greater than the earthquake input energy, structure can effectively resist the earthquake, and does not produce collapse. In the same time, Dr. George Housner explicitly proposed the concept of energy. The concept can better reflect the seismic intensity and spectral characteristics, and capture the inelastic deformation process of the structure under the action of strong earthquake from the input energy and dissipation of energy. The study of energy method becomes an important development direction to improve the traditional seismic design method. In the 1980s, Japan's Akiyama systematically researched the energy-based seismic design method. Based on the research results of the SDOF system and the MDOF system, the idea and method of energy-based seismic design were proposed, and some of them were applied in the Japanese seismic code. From the late 1980s to the 1990s, Fajfar studied the ground motion intensity index and the energy input and distribution relationship of the SDOF system, and proposed the N2 design method for the concrete structure. After 1990s, Fajfar and other scholars carried out a large number of studies on the seismic design method of the comprehensive consideration of the cumulative hysteretic energy dissipation and deformation. After 2000, Chou and Shen established the energy-based seismic design method of steel frame structure, and put forward the practical design flow.

Since the 1980s, Chinese scholars have begun to study energy-based seismic design methods. China's scholars have done a lot of research work in the energy input, distribution and the law of cumulative hysteretic energy dissipation of the SDOF system and MDOF system and the test of energy dissipation capacity of the structure and components. After many efforts of researchers, the foundation work of the energy law has been tending to improve, the corresponding design framework is basically mature, but there is no systematic design method.

2. Methods

2.1 Energy method theory

2.1.1 Expression of energy relation of single degree of freedom system

For an idealized single-layer structure affected by horizontal seismic motion, it can be regarded as an ideal single-degree-of-freedom system. The quality of the system focus on a point of the end, the single degree of freedom or the multi-degree of freedom system will have the damping, and dissipate the structural energy. If it is assumed that the axial deformation does not occur at the end, the system will have three degrees of freedom in the analysis of the static force, namely: the horizontal line displacement and the rotation angle of the two nodes. In the dynamic analysis under horizontal ground motion, the single-degree-of-freedom system requires only an independent horizontal displacement to determine the position of the particle under the action of the horizontal inertial force. Therefore, the system has only one lateral displacement degree of freedom. The displacement \( u' \) of the particle relative to the original rest position can be regarded as the superposition of two parts, which are the rigidity lateral displacement \( u_g \) of the whole structure and the horizontal displacement \( u \) of the particle relative to the structure base produced by the inertia force:

\[
u' = u_g + u
\]

Thus, it is easy to get a motion equation of single degree of freedom system, which can be expressed as:

\[
m\ddot{u}(t) + C\dot{u}(t) + f_s(u, \dot{u}) = -m\ddot{x}_g(t)
\]

(2)

\( C \) is the viscous damping coefficient; \( f_s(u, \dot{u}) \) is the restoring force of the structure;

When the structure is still in the linear elastic deformation stage, the restoring force of the system can be expressed by the following formula:

\[
f_s = Ku
\]

(3)

\( K \) is the initial lateral stiffness of the system. \( u \) is the relative displacement of the system;

The initial stiffness of the structure is represented by the stiffness of the beam, column and wall which the structure belongs, and can be obtained by a certain algebraic calculation. Under the action of rare earthquakes, the deformation of the structure may be in the elastic-plastic stage. In this process, the relationship between the restoring force and the displacement of the structure becomes very complex. In general, the restoring force will not correspond to the displacement value, but will depend on the deformation path and the deformation state, and can be expressed as:
\[ f_s = f(u, \dot{u}) \]

In the theoretical study, the general mathematical model of restoring force which is simplified on the basis of the experimental data is adopted. The equation (2) can be understood as the system substrate is stationary, and a horizontal equivalent force \( P = -m\ddot{u} \) is affected in the particle. Because the force is proportional to the quality of the system, the system quality is greater, the stronger the earthquake. The both ends of the motion differential equations (2) relatively displace \( u \) points to the particle, and the energy equation relative to the displacement can be obtained:

\[
\int_0^u m\dddot{u} du + \int_0^u C\dddot{u} du + \int_0^u f_s du = -\int_0^u m\ddot{u} g du
\]

Since the displacement \( u \) is a function of time \( t \), a differential relation \( du = u dt \) can be obtained, so that the integration of the displacement can be converted to the integration of time \( t \), and the above relationship can be substituted into (5) to obtain the expression of the seismic response energy calculation of the single degree of freedom system:

\[
\int_0^u m\dddot{u}(t) dt + \int_0^u C\dddot{u}(t) dt + \int_0^u f_s \dot{u} dt = -\int_0^u m\ddot{u} g (t) dt
\]

The equations on the left side of the above formula represent the different meanings, and the representing method is as follows:

\[ E_K = \int_0^u m\dddot{u} du = \int_0^u m\ddot{u}(t) dt \]

\[ E_D = \int_0^u C\dddot{u} du = \int_0^u C\dddot{u}(t) dt \]

\[ E_S + E_H = \int_0^u f_s du = \int_0^u f_s \dot{u}(t) dt \]

So at any time \( t \), the relationship of the dissipation energy balance in structure is:

\[ E_K + E_D = E_S + E_H = E_I \]

### 2.2.2 The expression of energy response of multi-degree-of-freedom system

In mathematics, because the structure is a continuous whole, there should be infinite degrees of freedom, but in specific engineering practice, the structure is usually equivalent to the limited multi-degree-of-freedom system to analyze according to certain rules, and the specific expression is:

\[
[M]\dddot{u}(t) + [C]\dddot{u}(t) + [R(t)] = [-M][r] \ddot{u}_g (t)
\]

\[ [M] \] is the diagonal matrix of the concentrated mass. \([C] \) is the damping matrix of the structure. \([R(t)] \) is the restoring force matrix of the structure. \( u(t) \) \( \dot{u}(t) \) \( \ddot{u}(t) \) respectively represent the displacement vector, the velocity vector and acceleration vector of the particle of the structure. \( \ddot{u}_g (t) \) is the acceleration of the ground motion; conversion column vector, and consider the structure of the damping matrix, \([r] \) is the switching column vector, and the corresponding terms of the degree of freedom in the direction of action of the seismic inertial force take 1, and the rest is 0.

When the earthquake continues to develop on the structure, the structure will enter the elastic-plastic stage, the stiffness matrix will change with time, and the specific performance is related to the position and the state of the each unit in their respective restoring force curve. Therefore, when solving differential equations, it is necessary to divide the whole seismic motion into a series of small steps of equal step length or different step length, and treat the structural parameters in each period as constants, and then use the stepwise integration method to solve.

In the structural analysis, the common step by step integration methods are linear acceleration method, the midpoint acceleration method, Markar \( \beta \) method and Wilson \( \theta \) method. In general, considering the aspects of stability and calculation accuracy of the model, the midpoint acceleration method or the Wilson's \( \theta \) method are used to analyze the nonlinear dynamics of the multi-degree-of-freedom system.

Similar to the energy response equation of the single-degree-of-freedom system, the general form of the energy-response equation of the multi-degree-of-freedom system can be expressed as a matrix:
The above equation can be abbreviated, and the simplified equation is:

\[
\int_0^T \{\ddot{u}(t)\}^T [M]\dddot{u}(t)dt + \int_0^T \{\ddot{u}(t)\}^T [C]\dddot{u}(t)dt + \int_0^T \{\ddot{u}(t)\}^T \{R(u(t))\}dt
\]

\[
= -\int_0^T \{\dot{u}(t)\}^T [M]\dot{\ddot{u}}_d dt
\]

(13)

The above equation can be abbreviated, and the simplified equation is:

Energy of the ground motion input structure:

\[
E_I(t) = -\int_0^T \{\dot{u}(t)\}^T [M]\dot{\ddot{u}}_d dt
\]

(15)

The kinetic energy of the structure:

\[
E_K(t) = \int_0^T \{\dot{u}(t)\}^T [M]\dot{\ddot{u}}(t)dt
\]

(16)

The damping dissipation energy of the structure:

\[
E_D(t) = \int_0^T \{\dot{u}(t)\}^T [C]\dot{\ddot{u}}(t)dt
\]

(17)

Elastic deformation energy and the hysteresis dissipation energy:

\[
E_S(t) + E_H(t) = \int_0^T \{\dot{u}(t)\}^T \{R(u(t))\}dt
\]

(18)

As the nonlinear dynamic analysis process is very complex, the calculation is very large. So, in general, in the actual design and analysis, it is necessary to do corresponding simplify and assumptions to the structure to have better results in the application.

The energy-based seismic design method first establishes the seismic input energy spectrum. The empirical criteria based on the numerical analysis are the equal energy criterion, the maximum deformation criterion, the instantaneous energy criterion and the equivalent linearization method in geometry. (1) The geometrical equivalent energy criterion considers that the elastic system and the elastic-plastic system are equal to the deformation energy obtained according to the geometrical calculation area in the uniaxial load-displacement curve. (2) The maximum deformation criterion considers that the elasticity system is almost equal to the maximum deformation of the elastic-plastic system, is an empirical conclusion obtained based on seismic response time history analysis results. (3) The instantaneous energy criterion is an effective method to predict the maximum response of the elastic-plastic system based on the energy concept. According to the input energy in the time interval close to the fundamental period of the building structure, the unidirectional maximum deformation is determined, that is, the instantaneous input energy is transformed into the energy dissipation of uniaxial load-displacement to predict the maximum deformation. (4) The equivalent linearization is a method suitable for the elastic system. It needs to assume that the damage distribution of the system is the same as that of the elastic system.

2.2 Establishment of analytical model of frame-shear structure

2.2.1 Structure model parameters

The engineering model is a part of the actual two-phase engineering under construction. The structure type adopts the cast-in-place reinforced concrete frame-shear wall structure, and the structure safety grade is the second level. The foundation design level is B level; the class of the building aseismicity is C-class, and the seismic fortification is 7 degree (0.15g). The site soil is Class II, and the design seismic grouping is the first group. The seismic rating framework of the special-shaped column-frame shear-wall structure is the three-tier level.

2.2.2 Structure model analysis

The basic modal analysis is carried out first to analyze the structural model, and the most original results of the structure are obtained, as shown in Table 1-3.

<table>
<thead>
<tr>
<th>Modal number</th>
<th>Frequency (Rad/sec)</th>
<th>Cycle (sec)</th>
<th>Allowable error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.7</td>
<td>1.0663</td>
<td>0.9378 0</td>
</tr>
<tr>
<td>2</td>
<td>7.7762</td>
<td>1.2376</td>
<td>0.808 0</td>
</tr>
<tr>
<td>3</td>
<td>8.1754</td>
<td>1.3012</td>
<td>0.7685 0</td>
</tr>
<tr>
<td>4</td>
<td>22.4687</td>
<td>3.576</td>
<td>0.2706 0</td>
</tr>
<tr>
<td>5</td>
<td>27.3295</td>
<td>4.3496</td>
<td>0.2299 0</td>
</tr>
<tr>
<td>6</td>
<td>28.2711</td>
<td>4.4995</td>
<td>0.2222 0</td>
</tr>
<tr>
<td>7</td>
<td>44.6431</td>
<td>7.1052</td>
<td>0.1407 2.29E-82</td>
</tr>
<tr>
<td>8</td>
<td>56.8744</td>
<td>9.0518</td>
<td>0.1105 1.76E-68</td>
</tr>
</tbody>
</table>
Table 2: Parameters and quality of the mode of vibration

<table>
<thead>
<tr>
<th>Modal number</th>
<th>TRAN-X</th>
<th>Total (%)</th>
<th>TRAN-Y</th>
<th>Total (%)</th>
<th>TRAN-Z</th>
<th>Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71.0445</td>
<td>0.1332</td>
<td>0.1332</td>
<td>0.0001</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.4741</td>
<td>67.3234</td>
<td>67.4566</td>
<td>0.0001</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.7839</td>
<td>3.652</td>
<td>71.1086</td>
<td>0.0001</td>
<td>0.0002</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14.1655</td>
<td>0.0082</td>
<td>71.1168</td>
<td>0.0001</td>
<td>0.0002</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0255</td>
<td>10.6149</td>
<td>81.7317</td>
<td>0.0025</td>
<td>0.0027</td>
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<tr>
<td>6</td>
<td>0.0097</td>
<td>3.9686</td>
<td>85.7003</td>
<td>0.0006</td>
<td>0.0033</td>
<td></td>
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<tr>
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<td>85.7009</td>
<td>0.0001</td>
<td>0.0034</td>
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<td>86.8046</td>
<td>0.0011</td>
<td>0.0046</td>
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<tr>
<td>9</td>
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<td>91.5417</td>
<td>0.001</td>
<td>0.0055</td>
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<tr>
<td>10</td>
<td>2.6458</td>
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<td>0.0058</td>
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<tr>
<td>11</td>
<td>0.0053</td>
<td>0.0726</td>
<td>91.6148</td>
<td>0.0002</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.0002</td>
<td>3.1349</td>
<td>94.7497</td>
<td>0.7585</td>
<td>0.7645</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Orientation factor of the mode of vibration

<table>
<thead>
<tr>
<th>Modal number</th>
<th>TRAN-X</th>
<th>TRAN-Y</th>
<th>TRAN-Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97.2574</td>
<td>0.1823</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.6695</td>
<td>95.0804</td>
<td>0.0001</td>
</tr>
<tr>
<td>3</td>
<td>2.4784</td>
<td>5.0737</td>
<td>0.0001</td>
</tr>
<tr>
<td>4</td>
<td>99.1613</td>
<td>0.0576</td>
<td>0.0006</td>
</tr>
<tr>
<td>5</td>
<td>0.1744</td>
<td>72.5288</td>
<td>0.0169</td>
</tr>
<tr>
<td>6</td>
<td>0.0686</td>
<td>28.0642</td>
<td>0.0043</td>
</tr>
<tr>
<td>7</td>
<td>99.9584</td>
<td>0.012</td>
<td>0.0017</td>
</tr>
<tr>
<td>8</td>
<td>0.0324</td>
<td>18.711</td>
<td>0.0194</td>
</tr>
<tr>
<td>9</td>
<td>0.0513</td>
<td>82.8047</td>
<td>0.0169</td>
</tr>
<tr>
<td>10</td>
<td>99.5879</td>
<td>0.0203</td>
<td>0.0115</td>
</tr>
<tr>
<td>11</td>
<td>0.1452</td>
<td>1.9918</td>
<td>0.0053</td>
</tr>
<tr>
<td>12</td>
<td>0.0049</td>
<td>73.7291</td>
<td>17.8393</td>
</tr>
</tbody>
</table>

The first mode of vibration of the structure is the translational motion in the X direction, the second mode is the translation in the Y direction, the third mode is the torsion, and the ratio of the first mode to the third mode is 73%, which meets less than 90% of the provisions. In the first mode and the second mode, the stress at the top of the structure is the largest, and in the third mode, the stress at the corner of the structure is the largest.

3. Results

3.1 Analysis of the displacement

Figure 1 Displacement of structure under the effects of Taft wave and EL wave on the frequent earthquakes
The results show that the average displacement of the whole structure is 14.15mm, which is in accordance with the specification. Comparing the floor displacement map, we can see that although the structure is subjected to different seismic waves, the response of the structure is similar. The structure is mainly composed of bending deformation in 1 – 4 layers, shear deformation in 5 – 8 layers and bending deformation in 9 – 11 layers. Under different seismic waves, the displacements and deformations of the structures are different. The displacements of the EL waves are much larger, which is determined by the structural dynamic characteristics and the spectral characteristics of seismic waves. Overall, the overall deformation of the structure shows bending and shearing state. Therefore, it can be deduced that the displacement of frame-shear-wall structure under different seismic waves is different, but the deformation is uniform.

3.2 Analysis of the structure energy

The input structure model of the Tianjin, Taft and EL waves under the rare earthquake are taken to calculate the energy response of the above structure, at the same time, the peak value of seismic wave acceleration was taken as 2209, and the seismic response time was calculated as 20s. The analysis results of the total energy time history is respectively shown in figure 3 and figure 4. From the figure, we can see that although the seismic wave intensity and the duration are the same, the energy response of the structure is also quite different. The total energy basically increases and the shock increases. The time history curve of the total energy increases continuously in the beginning, which corresponds to the concentrated area of the seismic wave intensity.

4. Discussion

From the comprehensive analysis of the effects of the frequent earthquake and rare earthquake on the structure, it can be concluded that the deformation curve of the whole structure under the action of multiple earthquakes is curved scissors type. And the displacement of the top layer of the structure is relatively large, because the mass and the rigidity setting is smaller. So the layout of the structure can not ignore the rational layout of the top structure. Under the action of rare earthquakes, the interlayer displacement value of the bottom layer of the structure shows the maximum value, and it is most likely to yield first.
Through comparison and analysis of the wave energy, the following conclusions can be drawn: 1) The total energy input of the structure under the action of rare earthquakes increases with oscillation, and the energy has a rapid growth section, which is usually located in the concentration section of the seismic wave intensity. 2) Because of the waveforms of the seismic waves and the structure, the results of this paper show that although the peak of acceleration and the duration of earthquake are the same, the energy response of the structure is very different. 3) Under the action of earthquake, the energy of the input structure is finally balanced by damping energy dissipation and non-elastic hysteretic energy dissipation. On the contrast of energy dissipation, the damping energy dissipation of the structure is smaller than that of the non-elastic hysteretic energy. 4) Under the action of Taft wave and EL wave, the input energy and energy dissipation of the earthquake will reach the maximum value quickly and will not increase again in the later period, which reflects that the response of the structure to earthquake is reduced, the strength and rigidity of the structure degrades, hysteresis loop area decreases.

5. Conclusion

Although we can not fully express the advantages of the energy law, there are still some shortcomings, but in general, the results of this energy analysis is consistent with the actual situation of the original project, and similar with the conclusions obtained by the existing methods, indicating that it has a certain reference value.

References