Event-Triggered Autonomous Platoon Control of Vehicles with Probabilistic Faults

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Abstract: This paper investigates event-triggered autonomous platoon control of vehicles with probabilistic faults. A novel even-triggered platoon model is established, in which the effect of probabilistic faults are involved. Based on the new model, criteria for co-designing both the even-trigger parameters and the controller gains are derived by using Lyapunov functional. The effectiveness of the presented methodology are demonstrated by experiments with laboratory scale Arduino cars.

Keywords: Platoon control, Even-triggered control, Probabilistic faults.

1. Introduction

The increasing demand for motor vehicles in today’s life brings huge burden on the existing ground transportation infrastructure. Therefore, a lot of research works on platoon control have been widely studied 1-2, for which is the main means to increasing the throughput of traffic, reducing fuel consumption and air pollution 3-4.

There are two different architectures on platoon control which are extensively investigated in the literature, including predecessor following and predecessor and leader following. The architecture is called predecessor following if the control action on a particular vehicle depends on the information with the predecessor. It was shown that this architecture suffers from string instability 5-6. That is, the response of a disturbance on an individual vehicle will be amplified along the string of vehicles. Alternatively, another control architecture investigated in the literature, and on which we focus in this research is predecessor and leader following platoon control structure. This control scheme is advantageous because, apart from its simplicity in achieving string stability, it utilizes the wireless communication technology to increase the performance of the platoon 7.

It is worth noting that most existing results on the predecessor and leader following control are limited in the following two aspects. Firstly, ignoring the frequently operating on the controller, this can increase fuel consumption. In 8, a model predictive control method was proposed to minimize the frequent operation on the brake or the throttle. Nevertheless, this control method suggested is not applicable to the predecessor and leader following platoon system. Secondly, without considering the probabilistic faults might happen to sensors and actuators in practical cars for reasons such as poor visibility due to rain, sandstorm and low battery power. In 9 the vulnerability of automobile radar sensors to weather phenomena such as rain and snow was studied. An actuator fault detection and fault tolerant control methodology are demonstrated by experiments with laboratory scale Arduino cars.

2. Experimental

2.1 Autonomous Platoon Modeling

Consider a vehicular platoon consisting of n vehicles. Denote \( z_i, v_i, a_i \) as the ith \((i=0,\ldots,n-1)\) vehicle’s position, velocity and acceleration, respectively. Define the spacing error of the ith following vehicle as:

\[
\delta_i = z_{i+1} - z_i - \delta_d - L_i ,
\]

where \( \delta_d \) is the desired vehicle spacing, \( L_i \) is the length of the vehicle. Then the dynamics of the ith following vehicle can be modeled by the following linear differential equations:

\[
\dot{a}_i(t) = -\frac{1}{\varsigma_i} a_i(t) + \frac{1}{\varsigma_i} u_i(t) \]

where \( \varsigma_i \) is the engine time constant, \( u_i \) is the control input signal to be designed.

Define \( x(t) = \text{Col}[x_i(t)]_{i=1}^{n-1} \), \( u(t) = \text{Col}[u_i(t)]_{i=1}^{n-1} \) and \( y(t) = \text{Col}[y_i(t)]_{i=1}^{n-1} \), respectively, as the state, the control and the measurement output vectors, where “Col” represents column vector, \( x_i(t) = [\delta_i, v_i, a_i]^T \) and \( y_i(t) = [\delta_i, v_{i+1} - v_i, a_{i+1} - a_i, v_0 - v_i, a_0 - a_i] \), \( i=1,\ldots,n-1 \). Based on (1) and (2), the state space equation of the entire platoon system can be written as:

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t),
\]

where \( A = \begin{bmatrix} A_0 & 0 & \cdots & 0 \\ A_1 & A_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & A_{n-1} & A_{n-2} \end{bmatrix}, B = \begin{bmatrix} B_0 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & B_{n-1} \end{bmatrix}, \quad A_{n-1} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\varsigma_i \end{bmatrix}, A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \).
For each following vehicle, the kernel controller can be designed as:

\[ u_i(t) = K_i y_i(t) \]  

(4)

where \( K_i = [k_p, k_v, k_s, k_u, k_w] \) is the controller gain to be determined.

Here, we introduce an event-triggered mechanism, which decides whether the newly sampled information should be sent to the controller. As is shown in Fig. 1, an event generator is constructed between the sensor and the controller, which decides when to transmit the measurement output to the controller by a specified trigger condition, the state is sampled regularly by the sampler of the sensor with period \( h \), and is fed into the event generator.

The state of the vehicles \( i \) is sampled at time \( k_h \) by sampler with a given period \( h \). The next state is at time \( (k+1)_h \). Considering the transmission delay is supposed to be \( \tau \), during the wireless transmission, the input of the controller in the time interval \( t_kh+\tau_k, t_{k+1}h+\tau_{k+1} \) can be described as

\[ u_kh \]  

Then, based on the above analysis, the controller in (4) can be rewritten as

\[ u(t) = KCCx(t_ih), \quad t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}] \]  

(6)

where \( K = \text{diag} \{K_i\} \) for each following vehicle.

Under the controller (6), the closed-loop platoon system for \( t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}] \) can be written in the following form:

\[ x(t) = Ax(t) + BKCCx(t_ih) \]  

(7)

For the convenience of forthcoming discussion, we convert system (7) into a time delay system. The holding interval of ZOH \( [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}] \) can be decomposed into the following subintervals

\[ [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}] = \bigcup_{l=0}^{i_{k+1}-i_k} \mathcal{I}_l \]  

(8)

where \( \mathcal{I}_l = [t_kh + \tau_k, i_{k+1}h + h + \tau_{k+1}] \) and \( i_kh = t_kh + lh \), \( l = 0, 1, \ldots, I_{k+1} - I_k - 1 \) means the sampling instants from the current controller active instant \( t_kh \) to the future controller active instant \( t_{k+1}h \). If \( l \) takes the value of \( I_{k+1} - I_k - 1 \), then \( \tau_{k+1} = \tau_{k+1} \), otherwise, \( \tau_k = t_{k+1}h \). Define

\[ \tau(t) = t - i_kh \]  

for \( t \in \mathcal{I}_l \), it is clear that \( \tau(t) \) satisfying

\[ 0 \leq \tau_m \leq \tau(t) \leq h + \tau_m \]  

Then, the closed system (7) can be written as
\( \dot{x}(t) = Ax(t) + BKCe(\tau(t)) + BKC(\sigma_1(t)) \) .

Now, we are in a position to considering the effects of sensor and actuator faults on the platoon systems. Firstly, we adopt the general sensor fault model in 11 to describe the fault phenomenon in GPS, wheel speed sensor and accelerometer. Taking sensors fault effects into consideration, we rewrite the controller (6) in the following form

\[
\begin{align*}
\dot{u}(t) &= KCp_x(x(t_1,h), t \in [t_{1\tau} + \tau_{1\tau}, t_{1\tau} + \tau_{1\tau} + \tau_{1\tau}]),
\end{align*}
\]

where \( \rho_{ac} = \text{diag}\{\rho_{ac}, \rho_{ac}, \ldots, \rho_{ac(n-1)}\} \) with \( \rho_{ac} = \text{diag}\{\rho_{ac}, \rho_{ac}, \rho_{ac}^{\Omega}\} \) and the mathematical expectation of \( \rho_{ac}^{\Omega} \) is \( \delta \). \( \beta_{ac}^{\Omega}, \beta_{ac}^{\Omega}, \beta_{ac}^{\Omega} \) denotes the faults states of GPS, wheel speed sensor and accelerometer, respectively.

Based on (11), we consider the actuator faults as in 12, then (7) can be described as

\[
\begin{align*}
\dot{u}(t) &= \rho_{ac} \times x(t_{1\tau}, h) t \in [t_{1\tau} + \tau_{1\tau}, t_{1\tau} + \tau_{1\tau} + \tau_{1\tau}],
\end{align*}
\]

where \( \rho_{ac} = \text{diag}\{\rho_{ac}, \rho_{ac}, \ldots, \rho_{ac(n-1)}\} \) represent the actuator fault state and \( 0 \leq \rho_{ac} \leq \rho_{ac}^{\Omega} \) with \( \rho_{ac}^{\Omega} \geq 1 \). The mathematical expectation of \( \rho_{ac}^{\Omega} \) is \( \beta_{ac}^{\Omega} \).

Based on the above analysis, the closed loop platoon system (10) can be rewritten as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + BKCe(\tau(t)) + BKCp_x(x(t_1,h), x(t) = \phi(t), t \in [-\tau, 0],
\end{align*}
\]

where \( \phi(t) \) is the initial function of the platoon system.

3. Controller Design

We first give the exponentially mean-square stable condition for the platoon system (13) in the following theorem.

**Theorem 1.** For given scalars \( \tau_{m}, \tau, \beta_{ac}^{\Omega}, \beta_{ac}^{\Omega}, \mu \in [0, 1] \) and feedback gain \( K \), the closed-loop platoon system in (13) is exponentially mean-square stable, if there exist matrices \( P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, R_3 > 0, \Omega > 0, N \) and \( M \) such that the following inequalities hold

\[
\begin{align*}
\Sigma = & \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} \\
\Sigma_{21} & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} \\
\Sigma_{31} & \Sigma_{32} & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} \\
\Sigma_{41} & \Sigma_{42} & \Sigma_{43} & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} \\
\Sigma_{51} & \Sigma_{52} & \Sigma_{53} & \Sigma_{54} & \Sigma_{55} & \Sigma_{56} \\
\Sigma_{61} & \Sigma_{62} & \Sigma_{63} & \Sigma_{64} & \Sigma_{65} & \Sigma_{66}
\end{bmatrix} < 0
\end{align*}
\]

where

\[
\Sigma_{11} = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & 0 & 0 \\
\Sigma_{21} & \Sigma_{22} & \Sigma_{23} & 0 & 0 & 0 \\
\Sigma_{31} & \Sigma_{32} & \Sigma_{33} & 0 & 0 & 0 \\
\Sigma_{41} & \Sigma_{42} & \Sigma_{43} & 0 & 0 & 0 \\
\Sigma_{51} & \Sigma_{52} & \Sigma_{53} & 0 & 0 & 0 \\
\Sigma_{61} & \Sigma_{62} & \Sigma_{63} & 0 & 0 & 0
\end{bmatrix}
\]

\[
\Sigma_{11} = P + A^TP + Q_1 + Q_2 - \frac{1}{\tau_{m}}(R_1 + R_2)^T
\]

\[
\Sigma_{12} = \begin{bmatrix}
\Sigma_{12} & 2P + \beta_{ac}^{\Omega} \beta_{ac}^{\Omega} KKC \Sigma_{12} & 1 \tau_{m} (R_1 + R_2)^T \\
\Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13}
\end{bmatrix}
\]

\[
\Sigma_{13} = \begin{bmatrix}
\Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13} & \Sigma_{13}
\end{bmatrix}
\]

\[
\Sigma_{14} = \begin{bmatrix}
\Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14} & \Sigma_{14}
\end{bmatrix}
\]

\[
\Sigma_{15} = \begin{bmatrix}
\Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15} & \Sigma_{15}
\end{bmatrix}
\]

\[
\Sigma_{16} = \begin{bmatrix}
\Pi_{1} & \Pi_{2} & \Pi_{3} & \Pi_{4} & \Pi_{5} & \Pi_{6}
\end{bmatrix}
\]

\[
\Pi_{1} = \begin{bmatrix}
\Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1} & \Pi_{1}
\end{bmatrix}
\]

\[
\Pi_{2} = \begin{bmatrix}
\Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2} & \Pi_{2}
\end{bmatrix}
\]

\[
\Pi_{3} = \begin{bmatrix}
\Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3} & \Pi_{3}
\end{bmatrix}
\]

\[
\Pi_{4} = \begin{bmatrix}
\Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4} & \Pi_{4}
\end{bmatrix}
\]

\[
\Pi_{5} = \begin{bmatrix}
\Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5} & \Pi_{5}
\end{bmatrix}
\]

\[
\Pi_{6} = \begin{bmatrix}
\Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6} & \Pi_{6}
\end{bmatrix}
\]
\[ \Sigma_{55} = \Sigma_{66} = \text{diag}\{-R^{-1}, -R^{-1}, \ldots, -R^{-1}\}, \quad \Gamma = [N \quad M - N \quad 0 \quad -M \quad 0]. \]

Proof: Define a Lyapunov function as
\[ V(x) = x^T(t)P(t)x(t) + \int_{t_0}^{t} \dot{x}(s)Q_{i}x(s)ds + \int_{t_0}^{t} \dot{x}(s)Q_{j}x(s)ds + \int_{t_0}^{t} \int_{t_0}^{s} \dot{x}(\theta)R_{i}\dot{x}(\theta)d\theta ds + \int_{t_0}^{t} \int_{t_0}^{s} \dot{x}(\theta)R_{j}\dot{x}(\theta)d\theta ds + 2\int_{t_0}^{t} \int_{t_0}^{s} \dot{x}(\theta)R_{i}\dot{x}(\theta)d\theta ds \]

(15)

where \( P, Q_{i} (i=1,2) \) and \( R_{j} (j=1,2,3) \) are positive-definite matrices.

Employing (15) and introducing some free matrices in 13, by using a similar method to the proof in 12, we can conclude that the system described by (13) is exponentially stable if (14) is satisfied. Due to page limitation, we omit the details in the proof.

Remark 2. Theorem 1 supplies a sufficient condition for the platoon system to be exponentially mean-square stable. We now proceed to give a reliable controller design method for system (13) based on Theorem 1.

Theorem 2. For given scalars \( \tau_{m}, \bar{\tau}, \beta_{iu}, \beta_{ij}, \varepsilon > 0 \) and \( M \in [0,1] \), the closed-loop platoon system in (13) with controller gain \( K \) is exponentially mean-square stable, if there exist matrices \( X > 0, \tilde{Q}_{i} > 0, \tilde{Q}_{j} > 0, \tilde{R}_{i} > 0, \tilde{R}_{j} > 0, \tilde{\Omega} > 0, \tilde{N}, M, \tilde{M} \) and \( L \) such that the following inequalities hold
\[ \begin{bmatrix} \tilde{\Sigma} & \tilde{\Theta} \\ \tilde{\Theta}^T & X \end{bmatrix} < 0 \]

(16)

where
\[ \tilde{\Sigma} = \begin{bmatrix} \tilde{\Theta}^T \Xi_{x} \Xi_{x}^T \Xi_{x} \\ \Xi_{x}^T \Xi_{x}^T \Xi_{x} \end{bmatrix}, \quad \tilde{\Theta}^T = [\tilde{\Theta}^T_{1}, \ldots, \tilde{\Theta}^T_{n-1}], \quad \Xi_{x} = [\Xi_{x1}, \ldots, \Xi_{x,n-1}], \quad \Xi_{x} = [\Xi_{x1}, \ldots, \Xi_{x,n-1}], \quad \Xi_{x} = [\Xi_{x1}, \ldots, \Xi_{x,n-1}]. \]

Furthermore, a candidate controller gain can be given by \( K = L X^{-1} \).

Proof: Defining \( X = P^{-1} \), then from (17), we can get
\[ \Sigma_{i} + \sum_{i=1}^{2} \sum_{j=1}^{n} \left[ \Theta_{i}^{T} \Theta_{i} + \Theta_{j}^{T} \Theta_{j} \right] \leq \Sigma_{i} + \sum_{i=1}^{2} \sum_{j=1}^{n} \left[ \Theta_{i}^{T} \Theta_{i} + \Theta_{j}^{T} \Theta_{j} \right] \]

(17)

where
\[ \Theta_{i}^{T} = [2(\varphi_{B\beta_{pq}}^{T} + \varphi_{B\beta_{pq}}^{T})], \quad \Sigma_{i} \text{ is obtained from } \Sigma \text{ by deleting } \Sigma_{i} \text{ and } \Sigma_{i} \text{, respectively.} \]

Furthermore, a candidate controller gain can be given by \( K = L X^{-1} \).
Combining (14), (17) and (18) and applying Schur complement, we can get

\[
\Sigma = \sum_{i=1}^{n} \left( \Theta_i X(\Theta_i) \right) \leq \sum_{i=1}^{n} \left[ \Xi_i + \Xi_i^{-1} \right] \leq \sum_{i=1}^{n} \left[ \Xi_i + \Xi_i^{-1} \right] = \sum_{i=1}^{n} \left[ \Xi_i + \Xi_i^{-1} \right]
\]

(19)

where

\[
\Sigma = \begin{bmatrix}
\Sigma_1 & \Theta_1 & \Xi_1 \\
\Theta_1 & \Xi_1 & \Xi_1 \\
\Xi_1 & \Xi_1 & \Xi_1
\end{bmatrix}
\]

\[
\Xi_1 = \begin{bmatrix}
\mu_1 & 0 & 0 \\
0 & \mu_2 & 0 \\
0 & 0 & \mu_3
\end{bmatrix}
\]

4. RESULTS AND DISCUSSION

The platoon system constitutes by five small-scale vehicles for the experiment are shown in Fig. 2. It is a radio control model car that has been transformed into a prototype autonomous vehicle. Atomega2560 is the main control unit. Two infrared sensors are used to measure the distance between two neighboring vehicles, which emulate the function of GPS. The model car is equipped with other accessories such as batteries, speed sensor, accelerometer and WiFi modular. The control algorithm is programmed in C language. The proposed event-triggered controllers have been realized by these vehicles. In the experiment, the sensors fault with the event-triggering scheme (5).
Wireless network
Vehicle 1
Sensor
Event generator
Controller
Actuator
Vehicle 0
Sensor
ZOH
Vehicle 2
Sensor
Event generator
Controller
Actuator
Vehicle n-1
Sensor
Event generator
Controller
Actuator

Fig. 1. Architecture of the event-triggered platoon system

(a)                                                                    (b)

Fig. 2, (a) Arduino car; (b) Arduino platoon system in experiment.

Fig. 3, The spacing and velocity characteristics for a four-vehicle platoon system under:
(a) Even-triggered controller (15); (b) Heterogeneous controller in 7.

Conclusions

In this paper, an event-triggered control scheme has been developed for autonomous platoon control of vehicles subject to probabilistic
faults. To eliminate the negative effect of the faults, an event-triggered control method was proposed. The experiments show the presented
method is in general superior to existing result.

References