Extended State Observer Based Compound Control for DC Motor Servo System without Speed Sensor

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Abstract: With regard to a typical DC motor servo system without actual angular speed sensor, a extended state observer (ESO) based compound control scheme is put forward in this paper. A nonlinear ESO (NESO) is designed to estimate the current uncertain dynamics and the actual angular speed of DC motor servo system, which improves the robust property of control system and lays the foundation for the design of outer-loop controller. Based on the as-built NESO in inner loop, an outer-loop compound controller by means of state-space design method is proposed in order to realize high-precision position tracking ability of DC motor servo system. Computer simulation results present that, compared with traditional control schemes, the proposed control scheme can guarantee the higher tracking precision of DC motor servo system. Moreover, it possesses stronger robustness against system uncertainties including modeling errors, parameter perturbations and friction moment disturbance, and also produces smooth control outputs, which is beneficial for the further implementation in engineering application.

Keywords: Active disturbance rejection control (ADRC), compound control, extended state observer (ESO), motor servo system, robustness.

1. Introduction

As a new magnetic material developed in recent years, the rare earth alloy has been successfully introduced to the production of modern brushless DC motor. Then, the response speed, the torque and the power/mass ratio of DC motor are all greatly enhanced [1]. As a result, the modern DC motors are widely applied in many high-precision position servo systems, such as the optic-electronic tracking system [2], the electro-mechanical actuator [3], the electronic flight simulator [4], and so on. At the same time, higher and higher performance requirements on DC motor servo control system are raised by the complicated and various working environment. However, because of the existence of system uncertainties in DC motor servo system, which usually involves modeling errors, parameters variations and disturbance torques, the performance development will be greatly restricted [5-6]. Therefore, it is necessary to explore the novel and useful control technologies for DC motor servo system, in order to further improve the robust property and tracking performance.

With regard to the general closed-loop feedback control system, it can be transformed into a compound control system by introducing one or more feedforward networks [13-15]. Through appropriately selecting control coefficients, the compound control system possesses high tracking accuracy and reliable stability. Subsequently, the compound control technology can solve the contradiction between reducing errors and maintaining stability in general feedback control system. Therefore, when it is applied to the servo system, the high speed and high accuracy of position tracking will be achieved [1]. As a result, the servo control scheme, which is constructed by the inner-loop ESO and the outer-loop compound controller, can not only suppress the bad impacts resulted from the uncertain dynamics in control system, but also improve the tracking performance of DC motor system. This is the main starting point of this paper. In addition, the feedback control needs the information of current angular speed, which can not be directly measured because of the lack of actual angular speed sensor, and then some estimation methods need to be employed to obtain the current angular speed. Note that, by using the measurable output signal, the above-mentioned NESO also has the ability to estimate the state variables of control plant accurately and real-time. Therefore, the introduction of inner-loop NESO can effectively deal with this problem. It means that the estimation of actual angular speed must be also realized in the construction of inner-loop NESO in this paper.

The remainder of this paper is organized as follows. Section 2 provides the mathematical modeling of DC motor servo system. In Section 3, the inner-loop NESO and the outer-loop compound controller are designed respectively, and then the whole servo controller is constructed. In Section 4, compared with traditional control schemes, the performance advantages of the proposed control scheme are verified through computer simulation. In addition, the conclusions and future works will be given in Section 5.

2. Mathematical modeling

A typical DC motor servo system is mainly composed by servo controller, amplifier, DC motor, loads and measurement unit, and its structure is shown in Figure 1.

Figure 1. Structural diagram of DC motor servo system.

Where, \( r \), \( u \) and \( y \) respectively denote the position angle command, the controller output and the current angle output of DC motor servo system. Note that, the part in dashed box is namely servo plant or control plant. Moreover, the measurement unit in Figure 1 is installed to merely measure the position angle signal, other than the angular speed signal.
Next, taking the external disturbances into consideration, and assuming that the mechanical connection between motor and loads is strictly rigid, the DC motor servo system can be described by the following differential equation [1]:

$$J \ddot{\theta}(t) + B \dot{\theta}(t) = u(t) + d(t) \tag{1}$$

where, parameters $J > 0$ and $B > 0$ respectively denote the equivalent moment of inertia and the equivalent damping ratio of the whole DC motor servo system. Moreover, $d$ denotes the equivalent disturbances transformed to the system input.

Because both the inner-loop NESO and the outer-loop compound controller proposed in this paper are established by means of state-space design methods, the further transformation about (1) will be carried out in next sequel.

Let state variables $x_1 = \theta(t)$ and $x_2 = \dot{\theta}(t)$ respectively denote the actual angle and the actual angular speed of DC motor servo system. As a result, Eq. (1) will be transformed into the following state-space expression:

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\frac{B}{J} x_2 + \frac{1}{J} u(t) + \frac{1}{J} d(t)
\end{cases} \tag{2}$$

Until now, the mathematical model of DC motor servo system has been established, and it provides the theoretical basis for the further design, analysis and verification.

3. Servo controller design

3.1 Inner-loop NESO design

If the current equivalent disturbances $d$ in system (2) can be effectively estimated by means of some useful observing strategies, the same amount of compensation will be introduced into the control input, which can realize the strong suppression against the bad impacts resulted from equivalent disturbances. Based on above-mentioned control thought, a type of NESO will be constructed for DC motor servo system (2), and it works as an inner-loop controller.

In actual engineering applications, the plant parameters $J$ and $B$ in (2) are unknown previously. We usually employ some measurement and identification methods to approximately obtain their values. As a result, the corresponding nominal parameters $J_n$ and $B_n$ are acquired and then used in the design of servo controller. Based on this statement, Eq. (2) will be transformed into the following expression:

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\frac{B}{J_n} x_2 + \frac{1}{J_n} u(t) + d_1(t)
\end{cases} \tag{3}$$

where, $d_1$ denotes a new uncertainty term, which involves modeling errors, parameter perturbations and external disturbances, and it is expressed by:

$$d_1(t) = \frac{J - J_n}{J_n} u(t) + \frac{J B_n - J_n B}{J_n} x_2 + \frac{1}{J} d(t) \tag{4}$$

Subsequently, $d_1$ is regarded as a extended state variable of system (3). Moreover, Eq. (4) provides the theoretical reference for the accuracy verification on uncertain dynamics estimation.

With regard to uncertain system (3), a NESO is proposed by using the measurable position output, and it is constructed as follows:

$$\begin{cases}
e = z_1 - x_1 \\
\dot{z}_1 = z_2 - \beta_0 e \\
\dot{z}_2 = z_3 - \beta_0 z_2 + \frac{1}{J_n} u(t) - \beta_0 \text{fall}(e, 0.5a, \delta) \\
\dot{z}_3 = -\beta_0 \text{fall}(e, 0.25a, \delta)
\end{cases} \tag{5}$$

where, the first two state variables ($z_1$ and $z_2$) of NESO are the estimation variables with respect to the angle ($x_1$) and the angular speed ($x_2$) respectively, meanwhile the last state variable ($z_3$) is the estimation variable with respect to extended variable ($d_1$).

Moreover, the parameters $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $a > 0$ and $\delta > 0$, the nonlinear function is defined as:

$$\text{fall}(e, \alpha, \delta) = \begin{cases}
|e| \geq \delta & \Rightarrow |e| = \delta \\
\frac{e}{\delta} & \text{for } |e| < \delta
\end{cases} \tag{6}$$

Therefore, the state variable $z_3$ will be regarded as the design reference of the compensation term in servo control system. Furthermore, the compensation term is designed as:

$$u_{in} = J_n z_3 \tag{7}$$

Then, the NESO-based inner loop control system is illustrated in Figure 2.
Servo Plant

\[ u_{out} \]

\[ + \]

\[ u_{in} \]

\[ J_s \]

NESO

\[ - \]

\[ inu \]

\[ out \]

\[ y \]

Figure 2. Structural diagram of NESO-based inner loop control system.

Where, \( u_{out} \) respectively denotes the output of outer-loop controller. Because the as-built NESO can only realize the robust performance of inner-loop control system, but cannot guarantee the position tracking function of DC motor servo system, therefore it is necessary to design another outer-loop controller based on position error adjustment, which will be introduced in the next subsection.

3.2 NESO-based compound controller

Considering the two-order dynamic characteristic of motor servo plant, we can design an outer-loop controller with compound structure, which is shown in Figure 3.

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -B_a/J_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J_a \end{bmatrix} u_{out} \]

Theorem 1 With regard to the NESO-based motor servo plant, the outer-loop compound control law can be designed as:

\[ u_{out} = k_1k_2 (r - \dot{y}) - k_2 \dot{y} + k_3k_4 \ddot{y} + k_3k_4 \dot{y} \]

where, \( \dot{r} \) and \( \ddot{r} \) respectively denote the angular speed command and angular acceleration command, \( \dot{y} \) denotes the current angular speed. If the following two conditions are both established,

1. \( k_1 \) and \( k_2 \) are designed in order that all the eigenvalues of matrix \( A_s - b u_{out} \) possess negative real parts, where \( k = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \).

2. \( k_3 \) and \( k_4 \) are designed according to the following relationship:

\[
\begin{align*}
    k_3 &= \frac{B_a}{J_a} + 1 \\
    k_4 &= \frac{k_2J_a}{B_a + k_2}
\end{align*}
\]

the position tracking ability of DC motor servo system will be realized, i.e., \( r \rightarrow y \).

Proof. Substituting the control law (9) into the system (8) gets:

\[
\begin{align*}
    \dot{x} &= A_s x + b u_{out} \left[ k_1k_2 (r - y) - k_2 \dot{y} + k_3k_4 \ddot{y} + k_3k_4 \dot{y} \right] \\
    &= A_s x + b u_{out} \left[ k_1k_2 (r - y) - b_a k_2 \dot{y} + b_k k_3k_4 \ddot{y} + b_k k_3k_4 \dot{y} \right] \\
    &= A_s x + b u_{out} \left[ (r - y) + k_2 (\dot{r} - \dot{y}) \right] + b_k k_4 \ddot{y} + b_k k_4 \dot{y} + b_k k_4 (k_3 - 1) \dot{y}
\end{align*}
\]
\[ x_e = x_d - x = [r - y \quad \dot{r} - \dot{y}]^T \]  

Then, Eq. (11) will be transformed into:
\[ \dot{x} = A_n x + b_n k x_x + b_n k_2 \dot{x} + b_n k_3 \left( k_3 - 1 \right) \dot{r} \]  

If the condition (2) is satisfied, Eq. (13) will be further rewritten by:
\[ \dot{x} = A_n x + b_n k x_x + b_n \left( J_n \ddot{r} + B_n \dot{r} \right) \Rightarrow \dot{x}_d - \dot{x}_e = A_n x + b_n k x_x + b_n \left( J_n \ddot{r} + B_n \dot{r} \right) \]  

The following equation can be easily obtained:
\[ \dot{x}_d - b_n \left( J_n \ddot{r} + B_n \dot{r} \right) = A_n x \]  

Substituting (15) into (14) gets:
\[ \dot{x}_e = -A_n x - b_n k x_x + A_n x_d = \left( A_n - b_n k \right) x_x \]  

Based on the poles assignment theory, because system (8) is totally controllable, we can design the feedback gain vector \( k \) to arbitrarily assign the eigenvalues of matrix \( A_n - b_n k \). That is, condition (1) can be easily met. As a result, the position tracking ability of DC motor servo system will be realized, i.e., Theorem 1 has been verified.

However, because of the absence of actual angular speed sensor, the current angular speed \( \dot{y} \) cannot be directly obtained. In this paper, the as-built NESO is employed to finish another task of observing the current angular speed. Therefore, the NESO-based compound control system is illustrated in Figure 4.

![Figure 4: Structural diagram of NESO-based compound control system.](image)

In conclusion, the total servo control law can be divided into two parts, and then expressed by:
\[ u(t) = u_{out} - u_m = k_1 k_2 \left( r - y \right) - k_2 k_4 \dot{z}_2 + k_2 k_3 \dot{r} + k_3 \ddot{z}_3 - J_n z_3 \]  

Until now, the servo controller of DC motor system has been completed, and its effectiveness will be verified through computer simulation in next section.

4. Simulation verification

In this section, in order to clearly present the performance advantages of the proposed control scheme in this paper, it is compared with the other two control schemes, which involve traditional PID control scheme and PD+NESO control scheme (i.e., removing the feedforward part from the proposed control scheme).

The actual parameters of servo plant are set as \( J = 0.0052 \text{N} \cdot \text{s}^2/\text{deg} \) and \( B = 0.065 \text{N} \cdot \text{s}/\text{deg} \). In order to conform with the actual condition in engineering application, some modeling errors should be taken into consideration in the design period. Therefore, the nominal model parameters of servo plant are chosen as \( J_n = 0.005 \text{N} \cdot \text{s}^2/\text{deg} \) and \( B_n = 0.06 \text{N} \cdot \text{s}/\text{deg} \). Moreover, we artificially add the Column friction torque and parameter perturbations into control system, and the equivalent disturbances resulted from the above two factors are respectively \(-4.0 \text{sign}(\dot{\theta}) \text{ (V)} \) and \(1.2 \sin(4\pi t + \pi/2) \text{ (V)} \). The initial angle and angular speed of DC motor servo system are respectively 0.2deg and 0deg/s. In addition, the outputs of servo controller are limited between \( \pm 10 \text{V} \).

In traditional PID control scheme, the three coefficients are chosen as:
\[ k_p = 5, \quad k_i = 2, \quad k_d = 0.2 \]

The coefficients of the inner-loop NESO are chosen as:
\[ \beta_0 = 1000, \quad \beta_1 = 10000, \quad \beta_3 = 1000000, \quad a = 1, \quad \delta = 0.01 \]

The coefficients of the outer-loop compound controller are chosen as:
\[ k_1 = 0.25, \quad k_2 = 0.02, \quad k_3 = 4, \quad k_4 = 0.0013 \]

In simulation, the angle command of DC motor servo system is chosen as a compound signal, which is shown in Figure 5.
With regard to the above command signal, the position tracking errors of DC motor servo systems under the three control schemes are shown in Figure 6.

From Figure 6, compared the conventional PID control scheme with PD+NESO control scheme, we can get that the tracking errors of DC motor servo system become smaller after introducing the inner-loop NESO, which reflects that the designed NESO effectively strengthens the robust property against uncertain dynamics. Then, compared PD+NESO control scheme with the proposed control scheme, we can find that the tracking errors of DC motor servo system become further smaller, which verifies the expected ability of outer-loop compound controller in improving the tracking accuracy of servo system.

In this paper, the inner-loop NESO is employed to estimate the current angular speed of DC motor and the uncertain dynamics in control system, therefore the accurate estimation is the basic guarantee for the effectiveness of NESO. In order to verify the estimation accuracy of the proposed NESO, the estimation curves with respect to angular speed and uncertain dynamics are shown in Figures 7-8.
From Figures 7-8, we can see that the state variables ($z_2$ and $z_3$) of the proposed NESO can accurately reflect the current values of angular speed and uncertain dynamics. Specially, the accurate estimation on the angular speed can improve the tracking properties (including tracking errors and regulation speed), when the outer-loop compound controller is used. Meanwhile, the accurate estimation on the uncertain dynamics can effectively strengthen the robustness of control system.

In addition, the control input of DC motor servo system is illustrated in Figure 9.

From Figure 9, we can see the control input of DC motor servo system is more smooth. This effectively avoids some bad impacts resulted from the high-chattering at control input, including system instability and damage to the mechanical structure of DC motor, and subsequently enhances the application value of the proposed control scheme.

5. Conclusion

With regard to the DC motor servo system without actual angular speed sensor, this paper puts forward a NESO-based compound control scheme, which can guarantee the better tracking performance of DC motor servo system. Specially, the inner-loop NESO can not only accurately estimate the unknown uncertain dynamics, i.e., the robust property of servo control system based on NESO compensation can be strengthened, but also accurately obtain the unknown angular speed, which guarantees the feasibility and the effectiveness of the outer-loop compound controller. Simulation results present that, compared with some traditional control schemes, the proposed one possesses more satisfactory tracking performance. Moreover, it is easy to be realized in engineering application. In addition, the experimental verification will be carried out in future works.

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References


